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NUMERICAL MODELING OF WATER ISOLATION IN THE DEVELOPMENT OF A LATERALLY HETEROGENEOUS OIL RESERVOIR

M.S. Khalilov[1], A.A. Karimova[1]

[1]Baku State University, Baku, Azerbaijan
e-mail: khaliov_mubariz@mail.ru , asudekerimova991@gmail.com

Abstract: The article proposes a two-phase (water, oil) and a three-component (water, oil, isolating component) numerical model that enables forecasting the efficiency of water inflow control in wells producing water-cut fluids at a specific stage of development of a reservoir that is laterally heterogeneous.Based on this model, a significant increase in oil recovery at any stage of field development is forecast by regulating the degree of water-flow isolation.

Keywords: Computational model, waterflooding, isolating component, degree of isolation, oil recovery factor.

AMS Subject Classification: 68U20.

1. Introduction

A low oil recovery factor is considered one of the main challenges in oil field development. At the same time, practical experience in reservoir development shows that high oil production rates in heterogeneous reservoirs lead to rapid advancement of formation water, edge water, or injected water, resulting in an increased water cut of the produced fluids [1-4,11].

In this regard, the problem of restricting water flow becomes particularly important. A considerable number of methods exist to address this problem through the use of plugging solutions, precipitation-forming agents, gels, wettability modifiers, and other technologies [6,12].

The results of mathematical modeling of water flow restriction using isolation compositions have been presented in a number of studies [7–9]. Despite the practical significance of these theoretical findings, several key issues remain unresolved, including the selection of the optimal timing for isolation considering the water cut of produced fluids; determination of the effect of the isolation degree (residual resistance factor); penetration depth of the water-blocking composition into the reservoir; duration of the isolation effect (or the washout time of the composition); and the possibility of accounting for capillary effects, among others.

In this regard, the development of a mathematical model capable of accounting for real reservoir conditions and, on this basis, evaluating the isolation efficiency of the blocking agent is of both scientific and practical interest.

2. Problem Formulation and Computational Scheme

The two-dimensional problem of oil displacement by water in a heterogeneous reservoir with a horizontal dip is considered, including the isolation of water inflow after water breakthrough at the production well, followed by the subsequent displacement of oil by water. Production and injection wells operate within the reservoir, and the external boundaries are assumed impermeable. The water injection rates at the injection wells are specified, while the volumetric production rates of oil and water at the production wells are calculated. The process of oil displacement and subsequent water-flow isolation is modeled using a two-phase (water–oil) and three-component (water–oil–isolating component or gel) filtration model. The model comprises the governing equations for two-phase flow, the mass balance equation describing the distribution of the gel within the reservoir to ensure water-flow isolation in the total phase flow, and the capillary pressure relationship governing the equilibrium of capillary forces between the phases [10,13].

$$\frac{\partial ms}{\partial t}+divV_w=\sum_{\nu=1}^{m_1}Q_{w\nu}^i(t)\delta(x-x_\nu^i,y-y_\nu^i)+\sum_{e=1}^{m_2}Q_{we}^p(t)\delta(x-x_e^p,y-y_e^p),$$

$$(x,y)\in D,t\in(0,T], \quad (1)$$

$$\frac{\partial m(1-s)}{\partial t}+divV_o=\sum_{e=1}^{m_2}Q_{oe}^p(t)\delta(x-x_e^p,\ y-y_e^p),\ Q_{oe}^p(t)=\begin{cases}Q_{oe}^0(t),t<T\\ Q_{oe}^i(t),t\geq T\end{cases},$$

$$(x,y)\in D,t\in(0,T], \quad (2)$$

$$\frac{\partial}{\partial t}\left[msc+m(1-s)\phi(c)+a(s,c)\right]+div(cV_w+\phi(c)V_o)=$$

$$=\sum_{e=1}^{m_2}(cQ_{we}^p(t)+\phi(c)Q_{oe}^p(t))\delta(x-x_e^p,y-y_e^p)+div\left[D(s,c)gradc\right],$$

$$c(x,y,t)=\begin{cases}0,t<T \quad before\ isolation\\ c_*(x,y,t),t\geq T \quad after\ isolation\end{cases} \quad (x,y)\in D,t\in(0,T],$$

(3)

$$p_o-p_w=p_{cr}(s,c), \quad (4)$$

$$s(x,y,t)\big|_{t=0}=s_0(x,y), \quad c(x,y,t)\big|_{t=0}=c_0(x,y)\ ,\ (0\leq x\leq l_x;\ 0\leq y\leq l_y),$$

(5)

$$\frac{\partial p_w}{\partial x}\Big|_{x=0,l_x}=0,\quad \frac{\partial p_o}{\partial x}\Big|_{x=0,l_x}=0,\quad \frac{\partial c}{\partial x}\Big|_{x=0,l_x}=0,\ \ 0\le y\le l_y,\ t\in(0,T]$$
$$\frac{\partial p_w}{\partial y}\Big|_{y=0,l_y}=0,\quad \frac{\partial p_o}{\partial y}\Big|_{y=0,l_y}=0,\quad \frac{\partial c}{\partial y}\Big|_{y=0,l_y}=0,\ \ 0\le x\le l_x,\ t\in(0,T]. \qquad (6)$$

Here, $V_w=-\frac{kf_w(s,c)}{\mu_w(p,c)}gradp_w,\ V_o=-\frac{kf_o(s,c)}{\mu_o(p,c)}gradp_o,$ denotes the filtration velocity of the water and oil phases; t - is time; T -is the time from the start of water injection until water breakthrough into the production well; x and y - are spatial coordinates; m - is porosity; s - is water saturation; c and $\varphi(c)$ - are the volumetric concentrations of the isolating component (blocking agent) in the water and oil phases, respectively; $a(s,c)$ - is the amount of isolating agent adsorbed per unit volume of the porous medium; m_1 and m_2 - are the numbers of injection and production wells, respectively; (x_ν^i, y_ν^i) and $(x_e^p,\ y_e^p)$ - are the coordinates of the injection and production wells; $Q_{w\nu}^i(t)$ - is the volumetric rate of water injected from the i-th injection well per unit reservoir thickness; $Q_{we}^p(t)$ and $Q_{oe}^p(t)$ - are the volumetric production rates of the water and oil phases, respectively, from the e-th production well per unit reservoir thickness; $Q_{oe}^p(t)$ - is the volumetric rate of the injected isolating component per unit time and per unit reservoir thickness at the e-th production well; $D(s,c)=(D_x(s,c),\ D_y(s,c))$ - is the vector of effective diffusion coefficients of the isolating component; k_i -is the absolute permeability of the porous medium; $f_w(s,c)$ and $f_o(s,c)$ - are the relative permeabilities of the water and oil phases in the porous medium;; $\mu_w(p,c),\ \mu_o(p,c)$ - are the viscosities of the water and oil phases; $p_w,\ p_o$ - are the pressures of the water and oil phases; $p_{cr}(s,c)$ -is the capillary pressure; D -is the flow domain of the fluids (water and oil). l_x and l_y -are the length and width of the reservoir, respectively.

In the filtration model, represented by the system of equations (1)–(6), the unknowns to be determined are the water saturation s, the concentration of the isolating component in the water phase c, and the pressure of the water phase p_w.

The iterative point-wise Jacobi method is applied to determine the pressure and the concentration of the isolating component, while the Euler method is used to compute the water saturation [5].

The function $Q_{fe}(t),\ \ (f=o,w)$, which accounts for the wells (water and oil production rates in the production wells), is modeled based on a quasi-steady two-phase flow with pressures $p_w(x_{iw}, y_{jw}, t)$ and $p_w(x_i, y_j, t)$ in internal regions bounded by circles of radius r_w and r_k. It can be written using [10,14] as:

$$Q_{fe}(t) = \overline{\varphi}(p,s,c)Q_e(t), \tag{7}$$

here $\overline{\varphi}_o(p_w,s,c) = \dfrac{\lambda_o(p,s,c)}{\lambda_o(p,s,c)+\lambda_w(p,s,c)}$, $\overline{\varphi}_w(p,s,c) = 1-\overline{\varphi}_o(p,s,c)$,

$$Q_e(t) = \frac{2\pi h}{\ln\dfrac{r_k}{r_w}} \int\limits_{p_w(x_{iw},y_{jw},t)}^{p_w(x_i,y_j,t)} \left[\lambda_o(p,s,c)+\lambda_w(p,s,c)\right]dp, r_k = \sqrt{\frac{\Delta x \Delta y}{\pi}}, \tag{8}$$

$$k\frac{f_w(s,c)}{\mu_w(p,c)} = \lambda_w, k\frac{f_o(s,c)}{\mu_o(p,c)} = \lambda_o,$$

$Q_e(t)$-is the total production (the cumulative water and oil production from the production wells); $\overline{\phi}_o$ and $\overline{\phi}_w$ -correspond to the respective amounts of oil and water in the flow. The volumetric flow rate $Q_e(t)$ per unit volume in the reservoir is defined as a single-valued quantity based on expression (8).

3. Calculation results.

The laterally extensive heterogeneous reservoir consists of two sections with different permeabilities. One production well and one injection well are located in the reservoir. They are positioned centrally along the vertical direction at the boundary line of the reservoir. Oil displacement is carried out by waterflooding under a constant hydrodynamic pressure drop of 2.4 MPa (Figure 1).

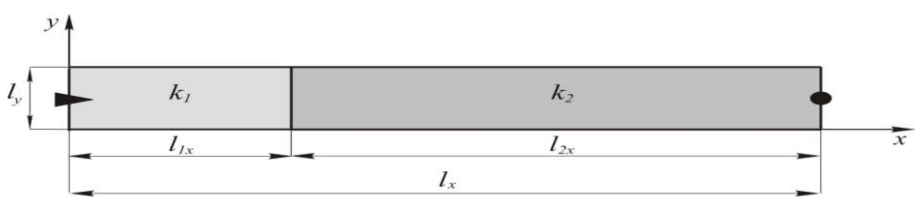

Figure 1. Schematic representation of the parameter distribution in a heterogeneous reservoir

The degree of isolation (residual resistance factor) is defined as the ratio of the phase permeability to water before and after water flow isolation:

$$k_{is} = \frac{f_w(s,c)}{f_{wai}(s,c)}$$

Here, k_{is} is the isolation coefficient (residual resistance factor); $f_w(s,c)$ is the relative phase permeability of water before isolation; $f_{wai}(s,c)$ is the relative phase permeability of water after isolation (determined using the identification method).

The following data were used for the reservoir parameters and the properties of the fluids:

$l_x = 500$ m; $l_y = 500$ m; $Q^i_{wv}(t) = 320$ m^3/day; $k_1 = 0.05 \cdot 10^{-12} - 10^{-12}$ m^2;

$k_2 = 0.05 \cdot 10^{-12} - 10^{-12}$ m^2;

$$s_0(x,y) = 0.2;\ \ c_0(x,y) = 0;\ \ a(s,c) = 0;\ \varphi(c) = c/2;$$

$\mu_w(p,c) = 1 + 0.55c - 0.3c^2$, mPa·sec; $\mu_o(p,c) = 14.6 - 14.7c + 5.5c^2$, mPa·sec

$$f_o(s,c) = \begin{cases} \left[\left(\dfrac{0.83-s}{0.834}\right)^3 - \left(\dfrac{0.74-s}{0.715}\right)^3\right]c + \left(\dfrac{0.74-s}{0.74}\right)^3, & 0 \le s \le 0.74 \\ \left(\dfrac{0.83-s}{0.834}\right)^3 c, & 0.74 \le s \le 0.83 \\ 0, & s \ge 0.83 \end{cases};$$

$$f_w(s,c) = \begin{cases} 0, & s \le 0.2 \\ \left(\dfrac{s-0.2}{0.81}\right)^3, & 0.2 \le s \le 1 \end{cases}$$

$$p_{cr}(s,c) = \Pi \cdot J(s);\quad J(s) = \frac{0.2(0.9-s)}{(1.2-s)^2(2s-s^2)};$$

$$D(s,c) = D_0 \cdot (2-s),\ \ \Pi = 10^5 \text{ Pa};\quad D_0 = 10^{-5} \text{ m}^2/\text{sec};$$

$$h_x = h_y = 20 \text{ m};\ \ c(x_e^p, y_e^p, T) = 0{,}05.$$

Based on the proposed computational model, the isolation process was investigated in a laterally extensive heterogeneous reservoir consisting of two sections with different permeabilities. The reservoir contains one injection well and one production well, located in the central part of the vertical plane along the reservoir boundary. Oil displacement was carried out by waterflooding under a constant hydrodynamic pressure drop of 2.4 MPa. The degree of isolation (residual resistance factor) is defined as the ratio of the water-phase permeabilities before and after the application of the water-flow isolation treatment.

Calculations were performed for two development scenarios:

Base scenario: The heterogeneous reservoir was developed under a waterflooding regime, with varying permeability and length ratios across the reservoir.

Water-flow isolation scenario: The reservoir was developed under a waterflooding regime. After water breakthrough at the production well, water flow was restricted at different degrees of isolation corresponding to various water-cut levels. The isolation process was carried out by injecting 60 m³ of a water-blocking agent into the production well.

For a heterogeneous reservoir with a single-layer permeability ratio varying across the field $(k_1 / k_2 = 0,05 \div 20)$ and different ratios of the lengths of the drainage areas of the injection and production wells $(l_{1x} / l_x = 0.25 \div 1)$, the water flow isolation process was performed for water cuts of 38% (low water cut), 50% (medium water cut), and 96% (high water cut). Isolation degrees (residual resistance factors) of 30, 60, and 90 were considered.

The calculation results are presented in Figures 2–9.

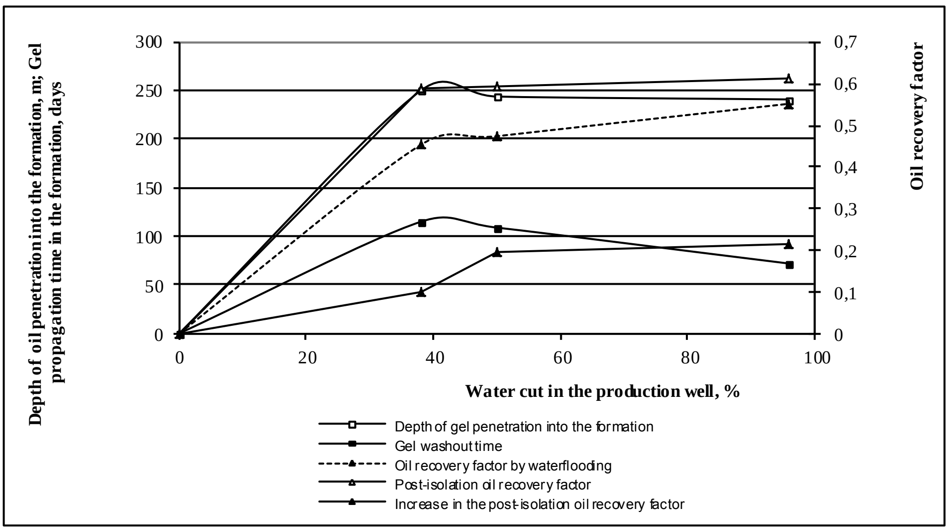

Figure 2. Dynamics of technological indicators in a heterogeneous reservoir in the $k_1 / k_2 = 0,05$ and $l_{1x} = 0,25\, l_x$ case with respect to water breakthrough in the production well

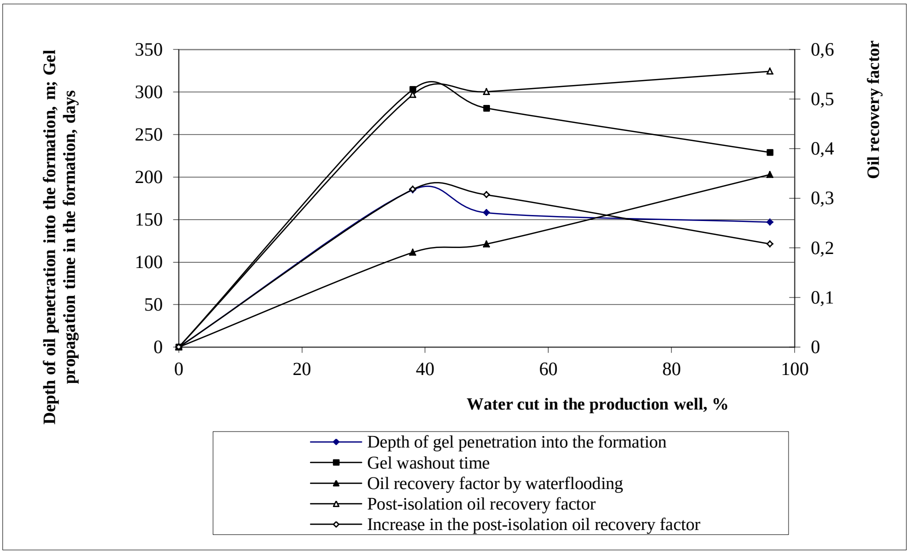

Figure 3. Dynamics of technological indicators in a heterogeneous reservoir for $k_1/k_2 = 20$ and $l_{1x} = 0{,}25\, l_x$ with respect to water breakthrough in the production well

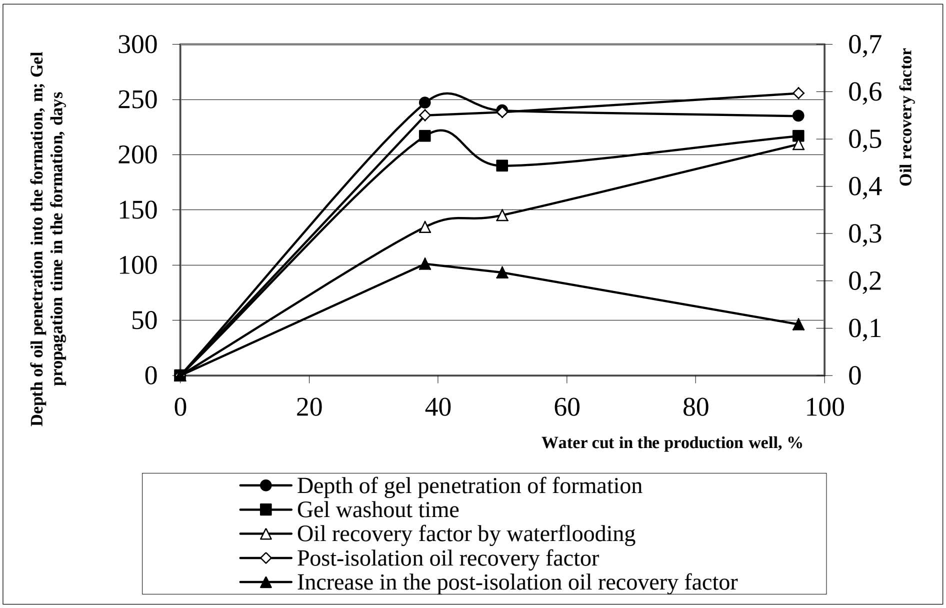

Figure 4. Dynamics of technological indicators in a heterogeneous reservoir for $k_1 / k_2 = 0.5$ and $l_{1x} = 0,5\, l_x$ with respect to water breakthrough in the production well

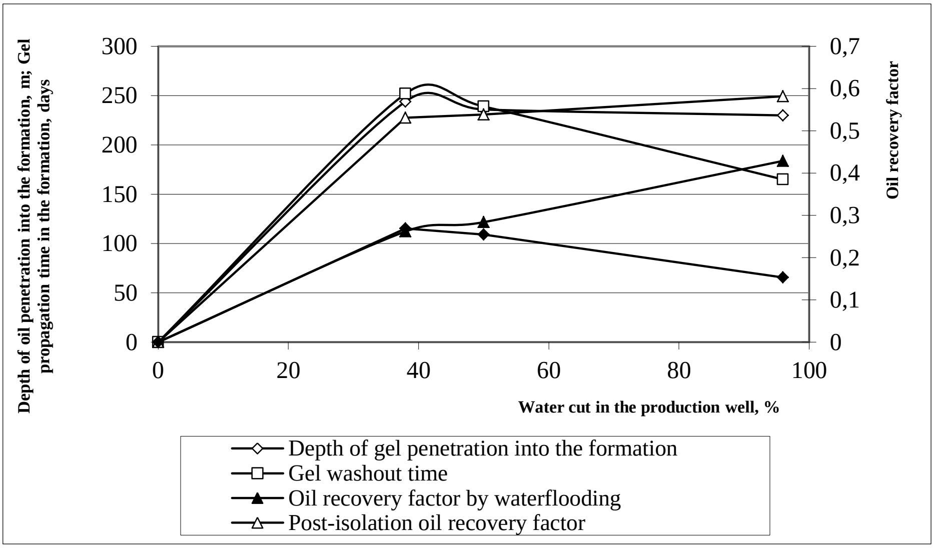

Figure 5. Dynamics of technological indicators in a heterogeneous reservoir for $k_1 / k_2 = 2$ and $l_{1x} = 0,5\, l_x$ with respect to water breakthrough in the production well

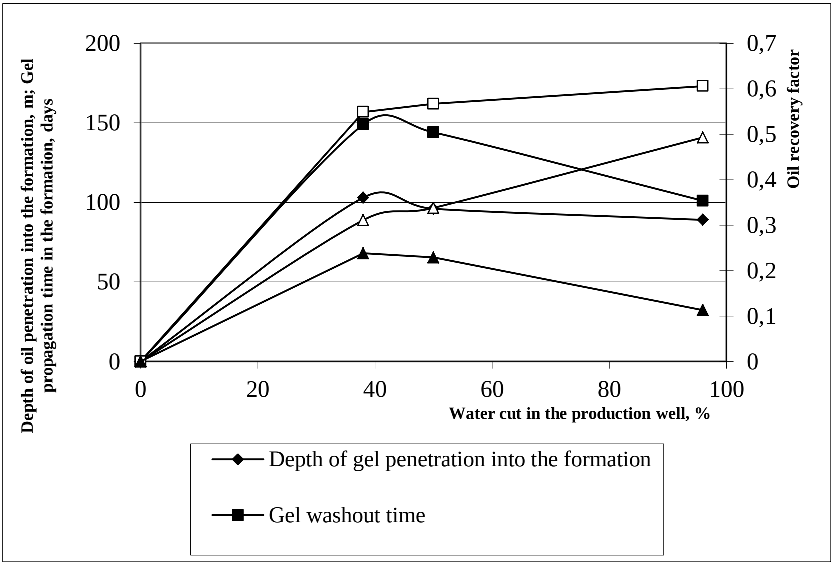

Figure 6. Dynamics of technological indicators in a heterogeneous reservoir for $k_1/k_2 = 0.05$ and $l_{1x} = 0,9l_x$ with respect to water breakthrough in the production well

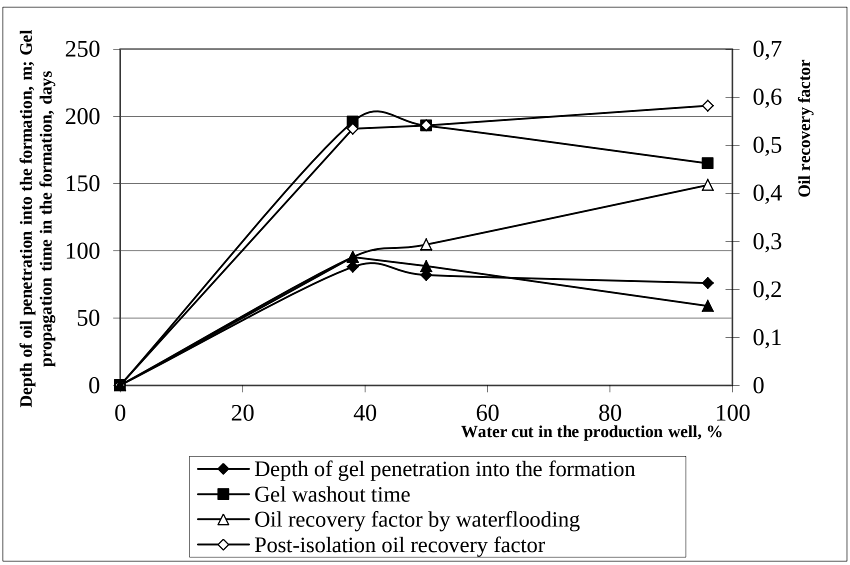

Figure 7. Dynamics of technological indicators in a heterogeneous reservoir for $k_1/k_2 = 20$ and $l_{1x} = 0,9l_x$ with respect to water breakthrough in the production well

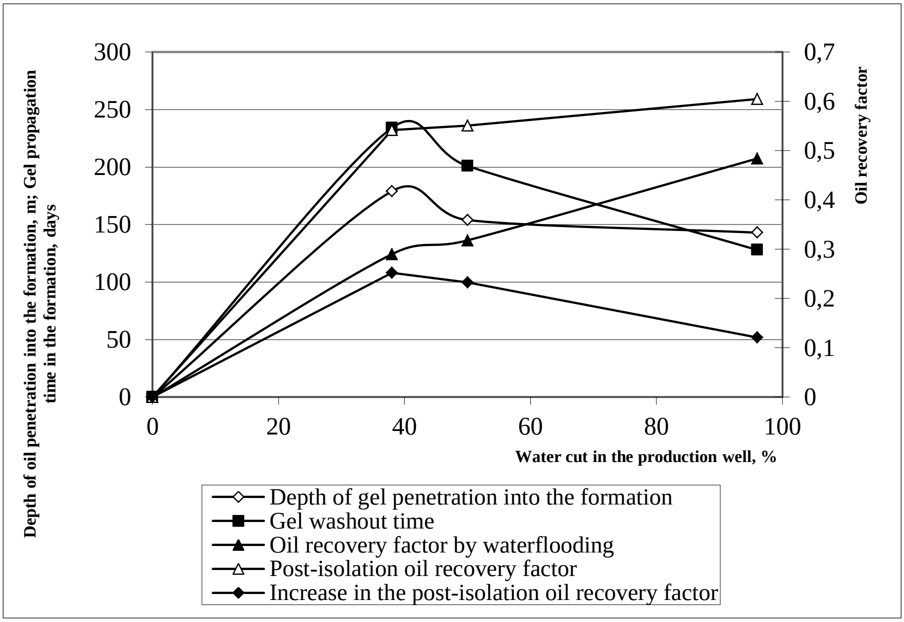

Figure 8. Dynamics of technological indicators in a homogeneous reservoir for $k = 0.05$ mkm^2 with respect to water breakthrough in the production well

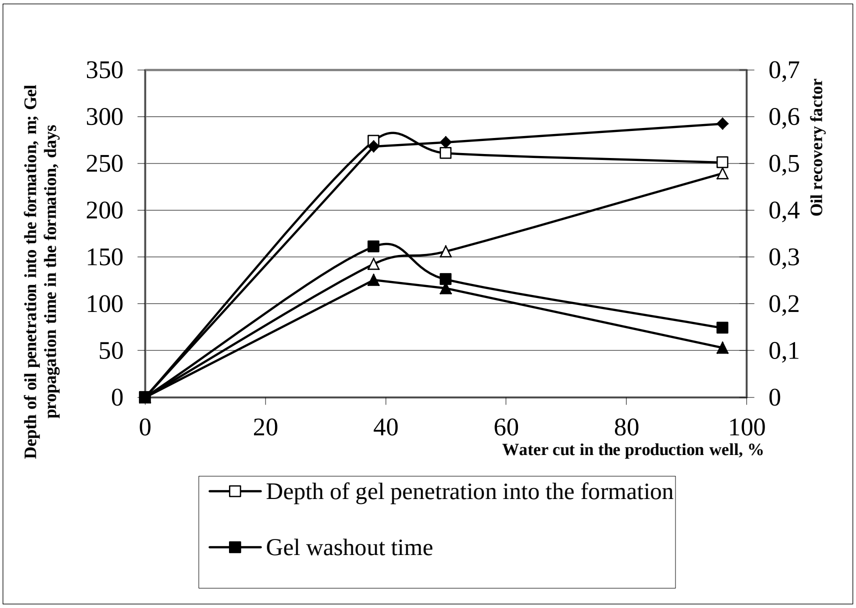

Figure 9. Dynamics of technological indicators in a homogeneous reservoir for $k = 2$ mkm^2 with respect to water breakthrough in the production well

The efficiency of the water flow isolation process was analyzed for a heterogeneous reservoir with a single-layer permeability ratio varying across the field $(k_1 / k_2 = 0,05 \div 20)$, where the lengths of the drainage areas of the injection and production wells varied within the $(l_{1x} / l_x = 0,25 \div 1)$ interval, and for cases with water cuts in the production well of 38% (low water cut), 50% (medium water cut), and 96% (high water cut).

In a low-permeability homogeneous layer, when the water cut at the production well is low, the penetration depth of the isolating agent near the wellbore increases, enhancing isolation effectiveness and prolonging its washout time. Similarly, a higher degree of isolation further improves its effectiveness. For example, at a 38% water cut, with isolation degrees of 30, 60, and 90, the effectiveness of isolation in terms of the current oil recovery factor increases from 0.252 to 0.289 compared to the waterflooding regime. As the water cut at the production well increases, the effectiveness of isolation decreases. For instance, when the water cut rises from 50% to 96%, the effectiveness in terms of the current oil recovery factor decreases on average from 0.25 to 0.124 relative to the 38% water cut case. Similar trends are observed in high-permeability homogeneous layers.

In a heterogeneous reservoir for the $l_{1x} = 0,25\ l_x$ case, when the permeability ratio is low ($k_1 / k_2 = 0,05$ or $k_1 / k_2 = 0,5$), at low water cut in the production well, the penetration distance of the isolating agent injected into the wellbore area increases compared to the corresponding results for higher water cuts, although these distances differ only slightly. The effectiveness of isolation in terms of the current oil recovery factor, compared to the waterflooding regime, increases on average as follows: 0.16, 0.14, 0.08 or 0.245, 0.23, 0.117, depending on the scenario. However, for isolation at high water cuts, the current oil recovery factor remains relatively high. For example, after low, medium, and high water cuts, the current oil recovery factor around the production well during isolation is approximately 0.605, 0.61, 0.627 or 0.57, 0.58, 0.61, respectively. When the permeability ratio is high ($k_1 / k_2 = 2$ or $k_1 / k_2 = 20$), at low water cut in the production well, the penetration distance of the isolating agent into the reservoir depth at the wellbore area decreases compared to the corresponding results for low permeability ratios. In this case, the effectiveness of isolation in terms of the current oil recovery factor, compared to the waterflooding regime, increases on average as follows: 0.3, 0.28, 0.195 or 0.38, 0.37, 0.28. For both low and high permeability ratios, at any water cut, an increase in the degree of isolation results in a relatively noticeable increase in the current oil recovery factor.

In a heterogeneous reservoir, for the $l_{1x} = 0,9\, l_x$ case, when the permeability ratio is low ($k_1 / k_2 = 0,05$ or $k_1 / k_2 = 0,5$), at any water cut in the production well, the penetration distance of the isolating agent injected into the wellbore area decreases by approximately 2.5 or 2.8 times compared to the corresponding results for the $l_{1x} = 0{,}25 l_x$ case. The effectiveness of isolation in terms of the current oil recovery factor, compared to the waterflooding regime, increases on average as follows: 0.265, 0.24, 0.118 or 0.27, 0.25, 0.12. For increasing water cuts, the current oil recovery factor during isolation varies as 0.57, 0.575, 0.611 or 0.562, 0.57, 0.603.

Similar results are obtained when the permeability ratio is high ($k_1 / k_2 = 2$ or $k_1 / k_2 = 20$). However, for any water cut, the effectiveness of isolation is higher than in the case of a low permeability ratio. At the same time, the current oil recovery factor is relatively lower compared to the corresponding results for the low permeability ratio case.

The washing-out time of the isolation material decreases with increasing water cut for any permeability ratio in the $l_{1x} = 0{,}25 l_x$ and $l_{1x} = 0{,}9 l_x$ cases.

4. Conclusion

1. A computational model is proposed for the filtration of a two-phase, three-component fluid in homogeneous and heterogeneous horizontal reservoirs, which accounts for capillary effects of water flow isolation, the degree of isolation, and the penetration depth of the isolating agent.
2. In low-permeability homogeneous and heterogeneous reservoirs, for any low or high permeability ratio, at any water cut in the production well, as the penetration distance of the isolating agent into the reservoir depth increases, the washing-out time of the isolating agent, the degree of isolation, and its effectiveness increase, while the current oil recovery factor significantly increases compared to waterflooding.

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